

# Proposed Test of Relative Phase as Hidden Variable in Quantum Mechanics

Steven Peil

Received: 7 April 2011 / Accepted: 16 August 2012 / Published online: 29 August 2012  
© Springer Science+Business Media, LLC (outside the USA) 2012

**Abstract** We consider the possibility that the relative phase in quantum mechanics plays a role in determining measurement outcome and could therefore serve as a “hidden” variable. The Born rule for measurement equates the probability for a given outcome with the absolute square of the coefficient of the basis state, which by design removes the relative phase from the formulation. The value of this phase at the moment of measurement naturally averages out in an ensemble, which would prevent any dependence from being observed, and we show that conventional frequency-spectroscopy measurements on discrete quantum systems cannot be imposed at a specific phase due to a straightforward uncertainty relation. We lay out general conditions for imposing measurements at a specific value of the relative phase so that the possibility of its role as a hidden variable can be tested, and we discuss implementation for the specific case of an atomic two-state system with laser-induced fluorescence for measurement.

**Keywords** Quantum measurement · Measurement postulate · Born rule

## 1 Introduction

### 1.1 Problems with Quantum Measurement

Quantum theory prescribes probabilities for outcomes of measurements performed on microscopic systems. Coherent evolution of a system is governed by the Schrödinger equation, allowing determination of the time-dependent amplitudes and phases of the states characterizing the system; the amplitudes of the measurement-basis states

---

S. Peil (✉)  
United States Naval Observatory, 3450 Massachusetts Ave., Washington, DC 20392, USA  
e-mail: [steven.peil@usno.navy.mil](mailto:steven.peil@usno.navy.mil)

Report Documentation Page			Form Approved OMB No. 0704-0188		
Public reporting burden for the collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to a penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.					
1. REPORT DATE <b>AUG 2012</b>		2. REPORT TYPE		3. DATES COVERED <b>00-00-2012 to 00-00-2012</b>	
4. TITLE AND SUBTITLE <b>Proposed Test of Relative Phase as Hidden Variable in Quantum Mechanics</b>				5a. CONTRACT NUMBER	
				5b. GRANT NUMBER	
				5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S)				5d. PROJECT NUMBER	
				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) <b>United States Naval Observatory, 3450 Massachusetts Ave, Washington, DC, 20392</b>				8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)				10. SPONSOR/MONITOR'S ACRONYM(S)	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT <b>Approved for public release; distribution unlimited</b>					
13. SUPPLEMENTARY NOTES <b>Found Phys (2012) 42:1523?1533</b>					
14. ABSTRACT <b>We consider the possibility that the relative phase in quantum mechanics plays a role in determining measurement outcome and could therefore serve as a ?hidden? variable. The Born rule for measurement equates the probability for a given outcome with the absolute square of the coefficient of the basis state, which by design removes the relative phase from the formulation. The value of this phase at the moment of measurement naturally averages out in an ensemble, which would prevent any dependence from being observed, and we show that conventional frequency spectroscopy measurements on discrete quantum systems cannot be imposed at a specific phase due to a straightforward uncertainty relation. We lay out general conditions for imposing measurements at a specific value of the relative phase so that the possibility of its role as a hidden variable can be tested, and we discuss implementation for the specific case of an atomic two-state system with laser-induced fluorescence for measurement.</b>					
15. SUBJECT TERMS					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT <b>Same as Report (SAR)</b>	18. NUMBER OF PAGES <b>12</b>	19a. NAME OF RESPONSIBLE PERSON
a. REPORT <b>unclassified</b>	b. ABSTRACT <b>unclassified</b>	c. THIS PAGE <b>unclassified</b>			

at the time of measurement determine the probabilities for the corresponding outcomes. While this formulation has been remarkably successful, there are issues associated with quantum measurement that have existed from the development of the theory [1, 2].

First, the fact that the theory is probabilistic and is unable to determine the outcome of a measurement on an individual system has been troubling to some, leading to suggestions that the quantum description of a system is incomplete and that there could be “hidden variables” that might determine measurement outcome. Second, there is no known underlying origin for the Born rule determining measurement probabilities; it is a postulate of the theory. This is unlike statistical mechanics, in which probabilities can be explained in terms of deterministic classical behavior. Finally, the unitary evolution governed by the Schrödinger equation cannot result in the observed outcome of a measurement; at what level of microscopic description the non-unitary measurement process enters is ill-defined. This is commonly known as the (quantum) measurement problem.

Despite claims that the success of quantum mechanics implies that these foundational problems are of little consequence, many have emphasized the importance of resolving these issues, either by re-interpretation or modification of the theory. Modifications required to address these issues could have significant implications on efforts to unify gravity and quantum mechanics [2, 3].

## 1.2 Testable Modifications to Quantum Mechanics

Some interpretations of quantum mechanics, such as the many-worlds and consistent-histories formulations, try to address the quantum measurement problem and the origin of the Born rule [4, 5]. But different interpretations of quantum mechanics make identical predictions, so these alternate theories must be considered untestable. On the other hand, models attempting to resolve the measurement problem have been proposed that introduce new features and make predictions that differ from conventional quantum mechanics, making the proposed theories subject to verification. An example is the model of continuous spontaneous localization, which attempts to treat wave function collapse with a stochastic noise term added to the Schrödinger equation and which makes predictions that should be put to the test by experiments attempting to demonstrate coherence on systems of increasing size and complexity [6, 7]. In these efforts to address the measurement problem, the probabilistic nature of quantum mechanics is considered a postulate.

Similarly, for attempts to address nondeterminism in quantum mechanics, interpretations or even putative modifications which make no predictions different from conventional theory are untestable. The deBroglie-Bohm pilot-wave theory is subject to this criticism [8]. Bell’s theorem and related work enable tests of general classes of hidden-variable theories. These tests contrast predictions of quantum mechanics with those of theories including local hidden variables, at least as applied to measurements on individual members of correlated systems. All experiments so far indicate that any hidden-variables must be nonlocal. While tests of general properties of hidden-variables theories have been fruitful, we are unaware of any experimental tests for a specific, proposed hidden variable.

### 1.3 Relative Phase and Measurement Outcome

The only prediction of measurement outcomes possible in quantum mechanics is the likelihood for a certain outcome given by the square of the coefficient of the measurement-basis state. The relationship between measurement outcome and probability amplitudes dictated by the Born rule has been validated implicitly due to its ubiquity in quantum theory, but searches for dependence of measurement outcome on other parameters have been lacking.

For a two-state system, normalization constrains the measurement probabilities and only one independent parameter is involved in predicting measurement outcome, the population difference between the basis states. The relative phase is the only other independent parameter characterizing a two-state system, and Born's rule for deriving measurement probabilities eliminates the phase from the formulation by construction. Here we consider the possibility of the relative phase of a two-state system playing a role in determining measurement outcome and therefore acting as a hidden variable.

## 2 Relative Phase as Hidden Variable

The possibility of the relative phase as a hidden variable has not been ruled out explicitly—no direct tests have been carried out, nor implicitly—it is naturally averaged over in the measurement process, eliminating the signature of any possible role in measurement outcome. Yet, specifically engineering the measurement process so that it occurs at a specific value of the relative phase enables a search for a role as a hidden variable.<sup>1</sup>

### 2.1 Bell's Theorem

The traditional framework for considering hidden variables is based on the work of John Bell, who showed that quantum mechanics predicts measurements made on entangled particles can exhibit correlations that cannot be explained with local hidden variables [9]. Bell-type tests are typically applied to two particles entangled in a state such as

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle|1\rangle + e^{i\phi}|1\rangle|0\rangle). \quad (1)$$

This could represent, for example, two spin-1/2 systems in a singlet state for  $\phi = \pi$ . Measurements on a particle are made in one of several bases, and the measurements applied to each system are space-like separated, so that the basis for the measurement made on system 1 cannot be communicated to the measurer of system 2. The quantum correlations associated with the measurement outcomes therefore cannot be attributed to local hidden variables. All Bell-type tests to date are consistent with the results of traditional quantum mechanics and imply that any hidden variables that could exist must be nonlocal.

<sup>1</sup>We implicitly mean a specific value of  $\phi \bmod 2\pi$ .

The relative phase of a two-state system is not ruled out as a possible hidden variable by Bell's theorem because it is fundamentally not a local variable; if the system is in a superposition of widely separated states, the relative phase has to be considered nonlocal. Additionally, the framework for Bell's theorem does not directly apply. The relative phase is a property of a superposition state, and entanglement is not necessarily involved. For a state like that in Eq. (1), testing the possibility of phase as a hidden variable requires looking for a dependence of measurement outcome versus the value of  $\phi$  at the moment of measurement. However, because the measurement process significantly affects the relative phase, the value at the moment of measurement may be very different than the value in the original state.

## 2.2 Existing Experiments

Searching for a dependence of measurement outcome on the value of the relative phase at which measurement occurs is difficult. The value of the phase is affected by the measurement process, to the extent that the relative phase at the moment of measurement naturally averages out over an ensemble. A special technique for forcing measurement to occur at a specific, reproducible phase is required. These points are discussed in detail in Sect. 3, where we introduce a general, quantitative model of measurement on a two-state system; we emphasize the abruptness of decoherence compared to the phase evolution of the system, demonstrating that measurement at a specific value of phase is meaningful; we evaluate the phase evolution of the system introduced during the measurement process, showing via an uncertainty relation that the value of the relative phase at the moment of measurement naturally averages out over an ensemble; and we present measurement strategies that circumvent this uncertainty relation and impose measurement at a specific phase.

## 2.3 Relative Phase in Standard Quantum Mechanics

According to standard quantum mechanics, the relative phase of a superposition state has no effect on measurement outcome; the Born rule for deducing measurement probabilities eliminates the relative phase by construction through the squaring of the complex probability amplitudes. Rather, the phase serves as a record of coherent evolution, and it plays an important role in atomic clocks and other atomic interferometers. In clocks that rely on spectroscopy in which the atomic transition is probed coherently, the clock is designed as an interferometer [10]. The first interaction, or beam splitter, creates a population difference in the atomic sample and initiates a period of coherent phase evolution. The value of the relative phase after this evolution gets “locked in” by generating a phase-dependent population difference at the second beam splitter. This population difference gives the information on the atomic frequency required for clock operation.

The actual measurement of the population in the two clock states after the second beam splitter destroys the coherence in the system. For any method of state readout employed in atomic clocks, the measurement process naturally perturbs the relative phase to the extent that the measurement cannot be considered to occur at a specific value of the phase. (In fact, for atomic clocks, the phase precession is typically fast

enough that the duration of the measurement process is long compared to a cycle of phase evolution.) It is for this type of measurement that we are interested in considering if there is a dependence on the relative phase.

## 2.4 Examples of Phase-Dependent Measurement Outcome

If measurement outcome for a two-state system were to depend on the relative phase, the implication would be that the Born rule would be an approximation that is only correct when measurements are averaged over the relative phase.

Classical and semi-classical examples where measurement outcome is phase-dependent, though in a way that is subtle or difficult to implement, are perhaps suggestive. The strength of the electric field of a classical optical signal varies with a phase that evolves at a rate that can be on order of  $10^{15}$  Hz. Directly measuring the phase-sensitive field strength is impractical with optoelectronic devices, yet the phase manifests itself much more straightforwardly through interference effects. In the semiclassical vector model for adding two quantum angular momenta, component momenta  $\mathbf{j}_i$  are portrayed as precessing about the resultant  $\mathbf{J}$  (see, for example, [11]). For cases where there are final states with  $m_J = 0$ , the projections of the  $\mathbf{j}_i$  along the quantization axis are always opposite in sign and precess at a rate proportional to the interaction strength (faster for the case representing the triplet state,  $|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$ , associated with larger  $\mathbf{J}$ , than for the singlet state  $|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$ , associated with smaller  $\mathbf{J}$ ). Measuring the projections of the  $\mathbf{j}_i$  implies interrupting this precession, leaving the system in a configuration representing  $|\uparrow\downarrow\rangle$  or  $|\downarrow\uparrow\rangle$ , depending on the phase when measured.

## 3 Phase-Specific Quantum Measurement

### 3.1 General Measurement Model

A general, two-state quantum system can be characterized with just two independent parameters, the population difference and the relative phase between the two states. The state vector  $|\psi\rangle$  can be written as

$$|\psi\rangle = \alpha|0\rangle + e^{i\phi}\sqrt{1-\alpha^2}|1\rangle, \quad (2)$$

where the amplitude  $\alpha$  and relative phase  $\phi$  are real, and  $|0\rangle, |1\rangle$  are orthogonal basis states. For an isolated system, with energy difference  $E$  between the higher energy state  $|1\rangle$  and ground state  $|0\rangle$ , a superposition state evolves with a relative phase  $\phi = \frac{E}{\hbar}t$  and a period  $\tau_\phi = \frac{2\pi\hbar}{E}$ . The measurement postulate of quantum mechanics states that measuring the system in the  $\{|0\rangle, |1\rangle\}$  basis will result in state  $|0\rangle$  with a probability of  $\alpha^2$  (and state  $|1\rangle$  with probability  $1 - \alpha^2$ ). These measurement probabilities derive from the Born rule,  $P_x = |\langle x|\psi\rangle|^2$ , an expression which dismisses the phase factor  $e^{i\phi}$  between the basis states that represent potential measurement outcomes.

### 3.2 System, Reservoir Timescales

In order to carry out a measurement, the system must be coupled to a macroscopic reservoir that introduces irreversible evolution by damping the energy of state  $|1\rangle$  at a rate  $\Gamma_1$ . A quantum fluctuation can introduce correlations between the system and reservoir, which decay in a time  $\tau_c$  [12]. In terms of measurement on the two-state system, the vanishing correlations mark the point of irreversibility. The correlation time is roughly the inverse bandwidth of the reservoir and is very short for a macroscopic reservoir. In most cases,  $\tau_c$  is much shorter than any time for the quantum system to evolve, so that the measurement process takes place over a very short interval of time and a very small range of relative phases  $\Delta\phi$ , which we can take to be zero. This amounts to the Markov approximation for the reservoir.

A quantum fluctuation corresponds to measurement on the coherent two-state system via the presence or absence of spontaneous emission from the  $|1\rangle$  to  $|0\rangle$  transition.<sup>2</sup> The average time needed to acquire information from the system for a measurement is the lifetime of state  $|1\rangle$ ,  $1/\Gamma_1$ , which we call the measurement time  $\tau_m$ .<sup>3</sup> Once the system and reservoir are coupled (some amplitude in  $|1\rangle$ ), the probability of a quantum fluctuation resulting in measurement at time  $t$  is  $P(t) \propto e^{-t/\tau_m}$ ; beyond this, the specific time for a fluctuation to occur is unpredictable (Fig. 1(a)). A measurement can be applied at a specific  $\phi$ , then, in a system in which  $\tau_m$  is short compared to the period  $\tau_\phi$ .

### 3.3 Measurement at Specific Value of $\phi$

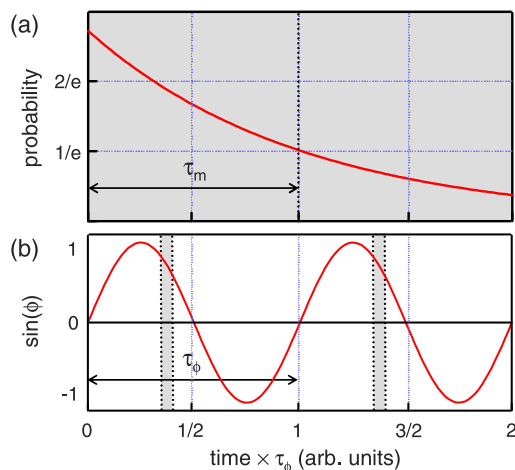
For systems where the phase evolution is very fast, such as in an atomic clock, this requirement on the measurement time is stringent. In fact, this condition is *never* satisfied, no matter how slow the phase evolution of the system, when the method used to differentiate the two states relies on resolving their frequency difference. The Fourier-transform limit for resolving a frequency difference  $\Delta\nu$  requires measuring for an interval of time  $\Delta t \geq 1/\Delta\nu$ . Since the energy difference between the states is  $E = 2\pi\hbar\Delta\nu$ , during this time the relative phase of a superposition state evolves by an amount

$$\frac{1}{\hbar} \int_0^{\Delta t} E dt' = 2\pi \Delta\nu \Delta t \geq 2\pi. \quad (3)$$

So the measurement time required to resolve two states spectroscopically is always on order of  $\tau_\phi$ , and measurement can occur at any value of  $\phi$  between 0 and  $2\pi$  radians. (The only phase specificity in this case is due to the exponential probability distribution for a measurement to occur.) We know of no measurements on two-state systems, including atomic clocks or specific qubits in quantum information, in which the phase of the state at the moment of measurement is not averaged out over an ensemble.

<sup>2</sup>See, for example, [13], for a discussion of state reduction via null measurement.

<sup>3</sup>More precisely, this could be called the minimum measurement time or the quantum measurement time. In the presence of technical noise, more averaging may be required to determine the state of the system.



**Fig. 1** (a) Probability of quantum fluctuation that entangles system and reservoir versus time after interaction is turned on. We call the average time for this distribution the measurement time  $\tau_m$ . Measurement can occur at any time within the *shaded area*. (b) Sine of relative phase  $\phi$  versus time. Measurement can be applied at a specific  $\phi = \frac{E}{\hbar} t$  by applying brief measurement pulses in synch with the phase evolution. These pulses—represented by the *shaded intervals*—are the only time the system-reservoir interaction is on and measurement is possible. The specific plots (a) and (b) correspond to the Fourier transform limit for a spectroscopic measurement, for which  $\tau_m = \tau_\phi$  (Color figure online)

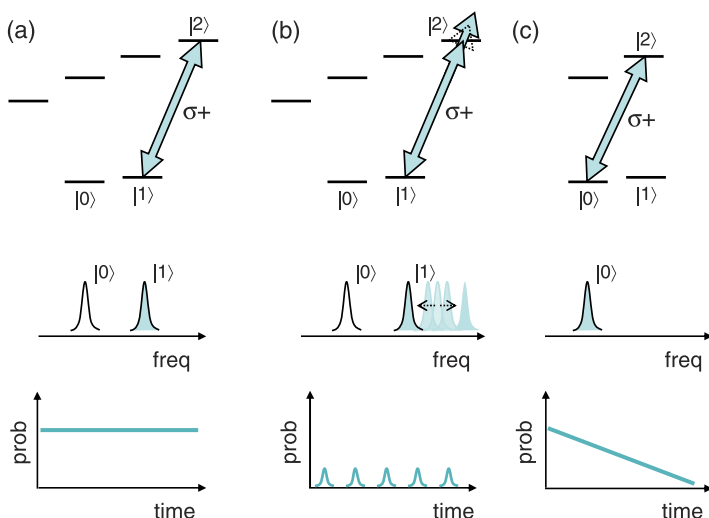
To get around this, a spectroscopic measurement can be applied in which the coupling to the reservoir is on only during a specific, narrow range of values of  $\phi$ , as illustrated in Fig. 1(b). This brief interaction, much shorter than  $\tau_m$ , is unlikely to result in measurement, but it can be repeated periodically, in synch with the coherent evolution of the phase, and after many such interactions the probability of measurement can approach unity. A second approach to realizing phase-selective measurement is to rely on a different method of state discrimination, such as using selection rules, in which case there is no Fourier-transform limit on the measurement time, allowing it to be much shorter than  $\tau_\phi$ .

#### 4 Application to Atomic System

We will discuss these examples in some detail for an atomic system. The two-state system can be realized with two long-lived electronic ground states. Different hyperfine levels typically have a frequency difference that is too large to satisfy  $\tau_m \ll \tau_\phi$ . However, Zeeman levels within a hyperfine manifold are degenerate at zero magnetic field and have a splitting that can be tuned with field. For low fields, the splitting between adjacent Zeeman sublevels is  $\Delta\nu = g\mu_B B$ , where  $g$  is the Lande  $g$ -factor for the states and  $\mu_B$  the Bohr magneton. For rubidium ( $^{87}\text{Rb}$ ), for example,  $\Delta\nu = 0.7 \text{ MHz/G}$  for the total angular momentum  $F = 2$  ground-state manifold.

In general, measurement is not carried out by looking for the decay  $|1\rangle \rightarrow |0\rangle$  as discussed above. For a naturally well isolated system, introduction of a third state,  $|2\rangle$ , which is more strongly coupled to its environment, enables more efficient and



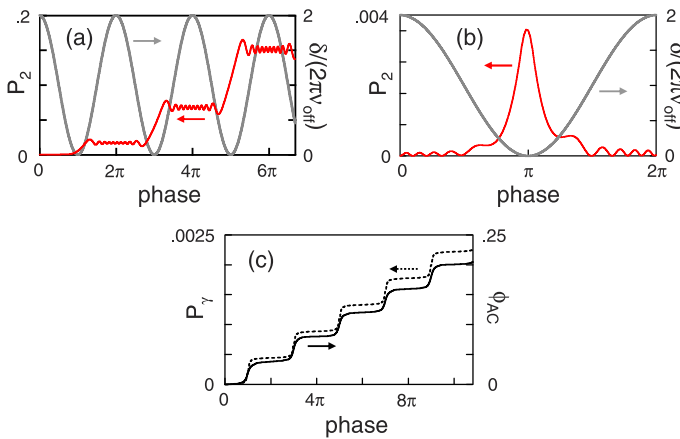


**Fig. 2** Energy level diagrams showing  $\sigma+$  transitions between two-state system and state  $|2\rangle$  (top), illustration of frequency of  $\sigma+$  transitions involving  $|0\rangle$  and  $|1\rangle$  (middle), and illustration of probability of scattering a photon versus time (bottom) for three scenarios. (a) Standard atomic-state detection technique relies on tuning a detection laser to be resonant with a specific transition. Circularly polarized detection light drives a closed  $\sigma+$  transition resulting in a constant average probability for an atom in state  $|1\rangle$  to scatter a photon. (b) In order to impose a measurement at a specific  $\phi$ , the frequency of the detection laser can be modulated synchronously with the phase evolution of the superposition state. By also adjusting the phase of the drive as discussed in the text, the probability of scattering a photon becomes a series of pulses. (c) For a detection transition between levels with the same angular momentum, selection rules can be used for state discrimination. Here, there are no cycling transitions, and the probability to scatter a photon decreases in time (Color figure online)

sensitive detection. For an atomic system, this third state is typically separated from the long-lived states by an optical frequency. Measurement in the  $\{|0\rangle, |1\rangle\}$  basis can be carried out by applying laser light that is tuned to the  $|1\rangle$  to  $|2\rangle$  transition. The optical drive coherently transfers amplitude to  $|2\rangle$ , which decays via spontaneous emission that can be detected as an indicator of population in state  $|1\rangle$ ; the absence of spontaneous emission indicates population in state  $|0\rangle$ . The emission process can be repeated many times if the transition is closed, i.e. if state  $|2\rangle$  can only decay to state  $|1\rangle$ ; this is illustrated in Fig. 2(a) for the case of circularly polarized light driving a  $\sigma+$  transition. Here, the measurement time for the  $\{|0\rangle, |1\rangle\}$  basis is the average time to scatter a photon on the  $|1\rangle \leftrightarrow |2\rangle$  transition,  $\tau_m = 1/(P_2\Gamma_2)$ , where  $\Gamma_2$  is the spontaneous emission rate and  $P_2$  the absolute square of the amplitude of state  $|2\rangle$ . The correlation time for the vacuum is less than a period of the optical frequency,  $\tau_c < 1/\nu_{21}$ , so the condition  $\tau_c \ll \tau_m$  is easily satisfied [12].

#### 4.1 Measurement Pulses

The first approach to engineering a phase-specific measurement requires generation of measurement “pulses,” brief intervals of time during which a quantum fluctuation resulting in measurement can occur. For our example, these pulses will be times when



**Fig. 3** Integration of the optical Bloch equations yields the population in the excited state  $|2\rangle$ ,  $P_2$ , due to the modulated detection laser. The calculations here are for the parameters  $\{2\pi\nu_{\text{off}}, 2\pi\nu_{\text{mod}}, \Omega_R, \Gamma_2\} = \{100, 10, 2, 1\}$ . **(a)** Plots of  $P_2$  (red curve, left axis) and  $\delta$  (grey curve, right axis) versus phase of modulation.  $P_2$  increases with each modulation cycle, and spontaneous emission can occur at any value of the phase of the two-state system. ( $P_2$  is calculated for  $\Gamma_2 = 0$ .) **(b)** Adjusting the phase of the detection drive at resonance can reverse the population transfer in the first half of the modulation cycle, leaving the atom in  $|1\rangle$ , and creating an interval outside of which the likelihood of measurement is small. Shown in the plot is  $\delta$  (grey curve) for one modulation cycle and  $P_2$  (red curve) for one measurement pulse. **(c)** Plots of probability of measurement (dashed curve, left axis) and accumulated phase on the  $|1\rangle \leftrightarrow |0\rangle$  transition from the AC Stark shift due to the detection laser (solid curve, right axis) versus phase of modulation. For this example, on order of 2000 pulses are required to ensure that a measurement occurs (Color figure online)

the detection laser can cause the atom to scatter a photon. Short intervals of resonant light can be created by frequency modulating the laser so that the detuning from the atomic transition is close to zero only for a small fraction of the modulation cycle (see illustration in Fig. 2(b)). We consider the detection laser frequency  $\nu_d$  centered above the atomic resonance,  $\nu_{21}$ , away from other nearby  $\sigma+$  transitions, and sinusoidally modulated so that it is resonant with the detection transition at an extremum. In terms of the detuning  $\delta = 2\pi(\nu_d - \nu_{21})$ ,

$$\delta = 2\pi\nu_{\text{off}}(1 + \cos(2\pi\nu_{\text{mod}}t)), \quad (4)$$

where the modulation frequency,  $\nu_{\text{mod}}$ , must match the frequency of the two-state system,  $\nu_{10}$ , to keep the detection pulses in synch with the phase evolution, and the offset frequency  $\nu_{\text{off}}$  then determines the fraction of the modulation cycle during which the laser can cause a photon to be scattered.

The brief intervals of resonant light do not constitute measurement pulses. A modulation cycle transfers some amplitude from  $|1\rangle$  to  $|2\rangle$ , as shown in Fig. 3(a). Because the probability of spontaneous emission is proportional to the square of the amplitude in the excited state, spontaneous emission can occur at any time and any phase—the only way to turn off spontaneous emission is to make the amplitude for state  $|2\rangle$  zero.

The interval of resonant light experienced by the atom during a modulation cycle can be converted into a measurement pulse by adjusting the phase of the optical drive halfway through the resonant interval to reverse the initial population transfer and

leave the atom in state  $|1\rangle$ , where no spontaneous emission can occur. For a constant detuning, reversing the population transfer would require a phase change of the optical drive of  $\pi$ ; the required change in the case of a particular frequency modulation can be determined empirically by integration of the optical Bloch equations. For the example in Fig. 3(b), the phase shift required is 0.75 radians. This adjustment to the phase can be implemented with an electro-optic modulator, which can be expected to require a time on order of 1 ns for a general phase change [14]. This imposes constraints on the duration of the detection pulse, and to make the pulse phase selective, on the period of evolution of the two-state system  $\tau_\phi$ ; the frequency splitting  $\nu_{10}$  has to be kept at or below about 50 MHz, which, for the rubidium example introduced earlier, corresponds to a magnetic field of 70 G.

Figure 3(c) shows the total probability that a measurement has occurred,  $P_\gamma$ , as a function of phase of the frequency modulation. The measurement pulse in Fig. 3(b) corresponds to a Rabi frequency  $\Omega_R$  on the  $|1\rangle$  to  $|2\rangle$  transition of  $2\Gamma_2$ , which leads to a 0.05 % probability of a photon being scattered (measurement being made) per measurement pulse. Because the frequency modulation is not symmetric about the detection transition, there is a finite AC Stark shift on the  $|0\rangle$  to  $|1\rangle$  transition of 0.05 radians per modulation cycle. This needs to be accounted for by modifying the modulation frequency so that the modulation cycle and atomic coherence are synchronized. For this example, the modulation frequency needs to be decreased by less than 1 %.

## 4.2 Selection Rules

The second scenario for imposing measurement at a specific phase is to use a method of state discrimination other than frequency measurement. Selection rules can be used to obtain a different response from the two atomic states,  $|0\rangle$  and  $|1\rangle$ , to the detection laser. In Fig. 2(c) energy levels are illustrated for a ground and excited state manifold that have the same angular momentum,  $F' = F$  ( $= 1/2$  in the illustration). If circularly polarized light is used to drive a  $\sigma+$  detection transition, only the  $|0\rangle$  to  $|2\rangle$  transition can be excited;  $|1\rangle$  does not couple to any excited state for this polarization. This method of state discrimination is not subject to the Fourier transform limit for measuring time—the existence or absence of scattered photons is sufficient to determine the atomic state, and therefore the measurement time can be much shorter than  $\tau_\phi$ . The magnetic field required to provide the quantization axis determines the frequency splitting of the two-state system and therefore  $\tau_\phi$ . The size of the field is limited only by the necessity to prevail over any stray fields that may be present and can be quite small, easily enabling  $\tau_m \ll \tau_\phi$ . For example, for the  $^{87}\text{Rb}$  transition at 795 nm,  $\tau_m \sim \tau_2 \sim 10$  ns, while  $\tau_\phi$  is 100  $\mu\text{s}$  for  $B = 10$  mG. The drawback of this approach is that the detection transition is not closed, and after scattering a small number of photons, the atom is optically pumped to  $|1\rangle$ , where it is “dark” to the detection drive. For equal Clebsch-Gordan coefficients for the transitions involved, the number of photons scattered before the atom is pumped into the dark state can be seen to be  $\sum_{n=1}^{\infty} n2^{-n} = 2$ . This is in contrast to thousands of photons that can easily be scattered on a closed transition without repump light.

## 5 Summary

In conclusion, we have discussed the possibility that the relative phase in quantum mechanics plays a role in measurement outcome. The value of the relative phase at the moment of measurement naturally averages out over an ensemble, and the difficulty in imposing a measurement at a specific  $\phi$  likely signifies that experiments have not been sensitive to any possible effect. We have presented two specific scenarios for forcing a measurement to occur at a specific phase, each of which should be achievable in current cold-atom systems.

## References

1. Wheeler, J.A., Zurek, W.H. (eds.): Quantum Theory and Measurement. Princeton University Press, Princeton (1983)
2. Adler, S.L.: Probability in orthodox quantum mechanics: probability as a postulate versus probability as an emergent phenomenon (2000). [quant-ph/0004077](https://arxiv.org/abs/quant-ph/0004077)
3. Smolin, L.: The Trouble with Physics, pp. 3–11. Houghton Mifflin Company, Boston (2007)
4. Everett, H. III: “Relative state” formulation of quantum mechanics. *Rev. Mod. Phys.* **29**, 454–462 (1957)
5. Griffiths, R.B.: Consistent Quantum Theory. Cambridge University Press, Cambridge (2002)
6. Ghirardi, G.C., Pearle, P., Rimini, A.: Markov processes in Hilbert space and continuous spontaneous localization of systems of identical particles. *Phys. Rev. A* **42**, 78–89 (1990)
7. Adler, S.L., Bassi, A.: Is quantum theory exact? *Science* **325**, 275–276 (2009)
8. Bohm, D.: A suggested interpretation of the quantum theory in terms of “hidden” variables. *Phys. Rev.* **85**, 166–179 (1952)
9. Bell, J.S.: On the Einstein-Podolsky-Rosen paradox. *Physics* **1**, 195–200 (1964)
10. Godun, R.M., D’Arcy, M.B., Summy, G.S., Burnett, K.: Prospects for atom interferometry. *Contemp. Phys.* **42**, 77–95 (2001)
11. Herzberg, G.: Atomic Spectra and Atomic Structure, pp. 82–119. Dover, New York (1944)
12. Cohen-Tannoudji, C., Dupont-Roc, J., Grynberg, G.: Atom-Photon Interactions, pp. 259–266. Wiley, New York (1992)
13. Porriati, M., Putterman, S.: Wave-function collapse due to null measurements: the origin of intermittent atomic fluorescence. *Phys. Rev. A* **36**, 929–932 (1987)
14. Mueller, H., Chiow, S., Herrmann, S., Chu, S.: Nanosecond electro-optical switching with a repetition rate above 20 MHz. *Rev. Sci. Instrum.* **78**, 124702 (2007)